

Assignment 13

This homework is due Tuesday May 3.

There are total 26 points in this assignment. 23 points is considered 100%. If you go over 23 points, you will get over 100% for this homework (but not over 115%) and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper *and give credit to your collaborators in your pledge*. Your solutions should exhibit your work and contain full proofs. Bare answers will not earn you much.

This assignment covers Sections 8.1–8.3 of Textbook.

RECALL that $C_\rho(z_0)$ denotes the circle of radius ρ centered at z_0 traversed counterclockwise.

- (1) [6pt] Find residue at 0 of the following functions. (There are different ways to do that, some may be easier than others in each particular case.) (*Hint*: In (1d), remember that $e^{a+b} = e^a e^b$.)

(a) $z^{-1}e^z$.	(c) $z^{-3}\cos z$.	(e) $\frac{e^{4z}-1}{\sin^2 z}$.
(b) $\frac{z^2+4z+5}{z^2+z}$.	(d) $e^{1+\frac{1}{z}}$.	(f) $z^{-1}\csc z$.

- (2) [6pt] Evaluate using residues:

(a) $\int_{C_1(1)} \frac{dz}{z^8-1}$. (Check carefully with points are inside the contour!)
(b) $\int_{C_2(0)} \frac{e^z dz}{z^3+z}$.
(c) $\int_{C_1(0)} \frac{dz}{z^2 \sin z}$.

- (3) [6pt] Use residues to find the following trigonometric integrals. (Don't forget that in each case the answer is a *real* number.)

In this problem you are allowed to use symbolic computation packages (for example, WolframAlpha¹) to compute residues.

(a) $\int_0^{2\pi} \frac{\sin^2 \theta}{5+4\cos \theta} d\theta$
(b) $\int_0^{2\pi} \frac{1}{(1+3\cos^2 \theta)^2} d\theta$

- (4) [8pt] Use residues to find the following improper integrals. (Don't forget that in each case the answer is a *real* number.)

In this problem you are allowed to use symbolic computation packages (for example, WolframAlpha) to compute residues. Be careful with output in 4b, though, since those packages are not always great with branching functions like roots.

(a) $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+16)^2}$.
(b) $\int_{-\infty}^{\infty} \frac{x^4 dx}{x^6+1}$.
(c) $\int_{-\infty}^{\infty} \frac{x+3}{(x^2+9)^2} dx$.
(d) $\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)}$, where $a > 0$, $b > 0$.

¹WA understands, for example, queries like “res $f(z)$ ” to list all residues of $f(z)$, or “res($f(z), a$)” to output residue of $f(z)$ at the point a .